Trunking Theory

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An Old Idea
- Trunking theory tells the size of a population that can be served by a limited number of servers with a specified grade-of-service (GOS)
- In the simple case, the GOS is the blocking probability
- Developed in the late 1800s by Erlang

Request Model
- Assume that customers request service at random times at a certain cumulative average rate, \( \lambda_{\text{cum}} \)
  - e.g. \( \lambda_{\text{cum}} = 13 \) requests per hour
- Times between consecutive requests are independent exponential random variables (RVs) with parameter \( \lambda_{\text{cum}} \)

Server Model
- The durations of service (i.e. the lengths of the calls or “holding times”) are independent exponential RVs with expected value \( H \)

Measuring Traffic Intensity
- An “Erlang” is the average fraction of time that a channel is occupied
- One continuous call is an example of traffic intensity of 1 Erlang
- A channel that carries traffic only half the time carries 0.5 Erlangs of traffic
- For the request and server models in previous slides, the traffic intensity is \( \lambda H \)

Single User vs. Total
- Suppose each user generates a traffic intensity of \( A_u \) Erlangs
- Suppose there are \( U \) users
- Then the total traffic intensity in Erlangs is \( A = A_u U \)
- Let \( \lambda \) be the rate of call requests per user. Then the traffic intensity per user can be expressed as \( A_u = \lambda H \)
Channels
- Each call requires a channel
- One approach is to dedicate a channel to each user
  - A user’s call request is never denied
  - A channel sits idle when its user is not making a call

  *Not an efficient use of resources!*

Trunked System
- Channels are “pooled”
- No user has a fixed channel
- A new user is assigned some channel from the pool
- When a call is finished, the channel is released back into the pool

“Block Calls Cleared”
- In a Block Calls Cleared type of system, a call request is simply denied if all channels in the pool are in use
- The blocked caller is free to make a new request
- Mobile Cellular systems are Block Calls Cleared systems

Probability of Blocking
- The GOS measure for Block Calls Cleared systems is the probability that a user’s call request is blocked
- The Erlang B formula determines the blocking probability, \( p \), given a certain total offered traffic intensity, \( A \), and a certain number of channels \( C \) in the pool

\[
p = B(A, C) = \frac{A^C}{\sum_{k=0}^{C} \frac{A^k}{k!}}
\]

Erland B Formula
- \( A \) is the total *offered* traffic
- Because some calls are blocked, \( A \) is not the traffic carried by the system

Trunking Gain
- Trunking gain is the improvement in offered traffic intensity that is obtained when sets of channels are merged into trunk pools
- In the next slide, the offered traffic intensities for a 10% blocking probability are compared for a \( C \)-channel trunked system and \( C \) fixed, single-channel systems
Summary

- In a trunked system, channels are pooled for common use on an as-needed basis.
- In a Block Calls Cleared system, a new request is simply denied if all channels are busy.
- The more channels in the pool, the higher the offered traffic can be for a given probability of blocking.

Graphical Comparison

Sketched from [Hernando and Pérez-Fontán, '99]

![Graphical comparison chart showing total offered traffic in Erlangs against the number of channels, with trunking gain and single-channel systems depicted.](image)

References