Frequency Shift Keying (FSK)

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Why Study FSK?
- Constant envelope
  - More efficient, less costly power amplifiers
- Gaussian minimum shift keying (GMSK), a special type of FSK, is used in the European digital cellular communications system (GSM)

Angle Modulation
- General form:
  - \( s(t) = A \cos(2\pi ft + \phi(t)) \)

Phase modulation (PM)
- \( \phi(t) = k_p m(t) \), where \( k_p \) = constant, \( m(t) \) = modulating signal
  - Examples: BPSK, QPSK, 8PSK, etc.

Frequency modulation (FM)
- \( \phi(t) = 2\pi \int_0^t k_f m(\tau) d\tau \), where \( k_f \) = constant, \( m(\tau) \) = modulating signal
  - Examples: FSK

Continuous Phase BFSK Illustration
- Phase continuity is important to reduce bandwidth

M-ary Continuous Phase FSK
- Recall FM phase: \( \phi(t) = 2\pi \sum k_n m(\tau) d\tau \)
  - The modulating waveform is pulse-amplitude modulated (PAM)
    - \( m(\tau) = \sum I_n g(\tau - nT_s) \)
    - where \( I_n \in \{\pm 1, \pm 3, \ldots, \pm (M - 1)\} \)
    - and \( g(\tau) \) is a pulse with area 1/2

Integrated Pulse
- Let \( q(t) \) be the integrated pulse
  - \( q(t) = \int_0^t g(\tau) d\tau \)
Modulation Index

- The FSK phase can be written
  \[ \phi(t) = 2\pi k, \sum_{n} f_{0}g(t-nT_{s}) \]
  - \( k \) is the modulation index \( k = 2f_{D}T_{s} \) with units of cycles/symbol period
  - \( f_{D} \) is the peak frequency deviation
  - The sum increments in multiples of 1/2

Example

- Suppose \( g(t) \) is the rectangular pulse, \( k_{j} = 10 \), and \( I_{n} = 1 \), then

Orthogonal Waveforms

- BFSK has the following waveforms:
  \[ s_{1}(t) = \sqrt{2E_{s}} \frac{\cos(2\pi(f_{c} + f_{D})t)}{T_{s}} \quad 0 < t < T_{s} \]
  \[ s_{2}(t) = \sqrt{2E_{s}} \frac{\cos(2\pi(f_{c} - f_{D})t)}{T_{s}} \quad 0 < t < T_{s} \]
- If \( f_{D} = n/4T_{s} \) for a positive integer, these waveforms will be orthogonal

BFSK Coherent Detection

\[ P_{\text{BFSK}} (\text{error}) = \frac{E_{s}}{2N_{0}} \]

\[ r(t) \bigtriangleup \cos(2\pi f_{c} + f_{D}, t) \]

\[ \int_{T_s} (\bigtriangleup t) \]

\[ \int_{T_s} (\bigtriangleup t) \]

\[ \text{Decision Circuit} \]

BFSK Noncoherent Detection

\[ P_{\text{BFSK}} (\text{error}) = \frac{1}{2} \exp \left( -\frac{E_{s}}{2N_{0}} \right) \]

Comparison

- In terms of BER,
  - CBFSK is to NBFSK as BPSK is to DPSK
  - The SNR for FSK is half as large as it is for PSK

\[ P_{\text{BPSK}} (\text{error}) = \frac{E_{s}}{N_{0}} \quad P_{\text{DPSK}} (\text{error}) = \frac{E_{s}}{N_{0}} \quad P_{\text{PSK}} (\text{error}) = \frac{1}{2} \exp \left( -\frac{E_{s}}{2N_{0}} \right) \]
MSK

- For minimum shift keying (MSK), the modulation index is as small as it can be and still yield orthogonal waveforms
- This minimum is $k_f = 1/2$
- For rectangular $g(t)$, the peak frequency deviation is $1/4$ the data rate

GMSK

- Gaussian MSK (GMSK) uses $k_f = 1/2$ and pulses related to the Gaussian shape
- GMSK pulses are distinguished by their time-bandwidth product $BT_s$
- $B$ is the 3-dB bandwidth of the pulse
- $BT_s = 0.3$ is used in GSM

Summary

- Frequency shift keying is a constant-envelope modulation technique
- Phase continuity reduces bandwidth
- FSK can be detected noncoherently
- GMSK, treated in detail in another module, is used in GSM

References