More on MSK

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**MSK is a Linear Modulation**
- MSK is an unusual type of FSK because it can be represented as a linear modulation
- It looks like OQPSK with pulses that are twice as wide
- The "symbols" weighting the pulses are not the original symbol sequence

**Recall BFSK**
- Recall the phase trajectories for binary FSK when rectangular pulses are used for $m(t)$:
  $$\phi(t) = 2\pi k_f \sum_n I_n q(t-nT_s)$$

The Phase for MSK
- In MSK, $k_f=0.5$
  $$\phi(t) = \pi \sum_n I_n q(t-nT_s)$$

**Y-intercept Form**
- The phase can be expressed
  $$\phi(t) = I_n \left( \frac{2\pi}{2T_s} \right) + x_n$$
  where $x_n$ is the Y-intercept for the $n$th symbol interval
  $$[x, \mod(2\pi)] \in [0,\pi]$$

**Example**
- Consider the symbol sequence $I_0=1$, $I_1=-1$, $I_2=-1$, $I_3=1$, $I_4=1$, $I_5=1$, $I_6=-1$
For $I_o=1$ and $0 < t < T_S$

$$\phi(t) = I_o \left( \frac{\pi}{2T_s} \right) + x_0 = 1 \left( \frac{\pi}{2T_s} \right) + 0 \quad \rightarrow \quad x_0 = 0$$

For $I_j=-1$ and $T_S < t < 2T_S$

$$\phi(t) = I_j \left( \frac{\pi}{2T_s} \right) + x_1 = -1 \left( \frac{\pi}{2T_s} \right) + \pi \quad \rightarrow \quad x_1 = \pi$$

For $I_2=-1$ and $2T_S < t < 3T_S$

$$\phi(t) = I_2 \left( \frac{\pi}{2T_s} \right) + x_2 = -1 \left( \frac{\pi}{2T_s} \right) + \pi \quad \rightarrow \quad x_2 = \pi$$

For $I_3=1$ and $3T_S < t < 4T_S$

$$\phi(t) = I_3 \left( \frac{\pi}{2T_s} \right) + x_3 = 1 \left( \frac{\pi}{2T_s} \right) - 2\pi \quad \rightarrow \quad x_3 = -2\pi = 0$$

I/Q Formulation of MSK

- It can be shown that

$$s(t) = \sqrt{\frac{2E_s}{T_s}} \left[ \sum_{n} c_n p(t - (2n-1)T_s) \cos(2\pi ft) - \sum_{n} d_n p(t - 2nT_s) \sin(2\pi ft) \right]$$

where

$$p(t) = \begin{cases} \sin \left( \frac{\pi t}{2T_s} \right) & 0 \leq t \leq 2T_s \\ 0 & \text{otherwise} \end{cases}$$

$$c_n = (-1)^n \cos x_{2n-1}$$

$$d_n = (-1)^n I_j \cos x_n$$

Example, cont’d

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<tr>
<th>$n$</th>
<th>$I_n$</th>
<th>$x_n \mod 2\pi$</th>
<th>$\cos x_n$</th>
<th>$c_n$</th>
<th>$d_n$</th>
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Baseband Waveform for the Example

MSK Resembles OQPSK
- The basic pulse shape is half a sinusoid that is $2T_b$ wide

Quadrature Detection
- The MSK receiver can be implemented coherently like an OQPSK receiver

Summary
- MSK is actually a linear modulation method
- The basic pulse shape is half a sinusoid that is two bit periods wide
- Resembles OQPSK with this basic pulse shape

References