Orthogonal Frequency Division Multiplexing (OFDM)

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OFDM Advantages
- OFDM is a spectrally efficient modulation technique
- It is conveniently implemented using IFFT and FFT operations
- It handles frequency selective channels well when combined with error correction coding

OFDM Disadvantages
- More complex than single-carrier modulation
- Requires a more linear power amplifier

OFDM Concept
- Suppose the symbol length is $T_s$
- Sinusoidal signals differing in frequency by $1/T_s$ will be orthogonal over the period $T_u$

$$\int_{t_o}^{t_o+T_u} e^{j2\pi f_o t} \left[ e^{-j2\pi \left( f_o + \frac{1}{T_u} \right) t} \right] dt = 0$$

Conceptual Transmitter Block Diagram

Subcarriers
- Each branch corresponds to a subcarrier
- Subcarriers are separated by $1/T_u$ Hz
- Each subcarrier modulates a different symbol
  - $b_k$ can be QAM
One OFDM Symbol

$$s(t) = \begin{cases} \sum_{k=0}^{N-1} b_k(t) e^{-j2\pi \left( f_s + \frac{k}{T_s} \right) t} & 0 < t < T_w \\ 0 & \text{otherwise} \end{cases}$$

Subcarrier Spectra

- Each modulated subcarrier has a spectrum in the shape of a sinc squared function

Cyclic Prefix Generation

- To avoid losing the power from echoes, a copy of the end is appended to the beginning of the “useful” part

Cyclic Prefix Appended

- The length of the cyclic prefix, also known as the Guard Interval, $D$, is supposed to be longer than the excess delay of the longest significant echo

DSRC/WAVE Timing-Related Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{dc}}$ number of data subcarriers</td>
<td>48</td>
</tr>
<tr>
<td>$N_{\text{pm}}$ number of pilot subcarriers</td>
<td>4</td>
</tr>
<tr>
<td>$f_0$ subcarrier frequency spacing</td>
<td>1 MHz/16 (16 MHz bandwidth)</td>
</tr>
<tr>
<td>$T_{\text{FFT}}$ FFT period</td>
<td>6.4 µs (16s)</td>
</tr>
<tr>
<td>$T_{\text{pulse}}$ pulse duration</td>
<td>32 µs (100 µs pulse)</td>
</tr>
<tr>
<td>$T_{\text{guard}}$ duration of the DIGITAL BPSK-OFDM period</td>
<td>8 µs ($T_{\text{guard}} = T_{\text{FFT}}$)</td>
</tr>
<tr>
<td>$T_{\text{DGI}}$ DGI duration</td>
<td>16 µs ($T_{\text{DGI}} = T_{\text{FFT}}$)</td>
</tr>
<tr>
<td>$T_{\text{short}}$ short training sequence duration</td>
<td>3.2 µs ($T_{\text{short}} = T_{\text{DGI}}$)</td>
</tr>
<tr>
<td>$T_{\text{long}}$ long training sequence duration</td>
<td>9.6 µs ($T_{\text{long}} = T_{\text{DGI}}$)</td>
</tr>
<tr>
<td>$T_{\text{preamble}}$ preamble duration</td>
<td>18 µs ($T_{\text{preamble}} = 2 \times T_{\text{DGI}}$)</td>
</tr>
</tbody>
</table>

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Effects of Multipath Delays

- A delayed echo of each subcarrier adds either constructively or destructively to its un-delayed version, creating a flat-faded version of that subcarrier

$$h(t, \tau)$$

All echoes completely fill the $T_s$ window
Cyclic Prefix Removal

- After the symbol has been received and stored in a buffer, the cyclic prefix is removed, leaving only the modulated and faded subcarriers.

Received OFDM Symbol, After Cyclic Prefix Has Been Removed

- For a static, frequency selective channel,

\[ r(t) = \left\{ \begin{array}{ll}
\sum_{u=0}^{N-1} b_u(t) H \left( f_u + \frac{k}{T_u} \right) e^{-j2\pi f_u t T_u} + n(t) & 0 < t < T_u \\
0 & \text{otherwise}
\end{array} \right. \]

FFT

- These correlations are conveniently performed by the Fast Fourier Transform (FFT).
- The modulation is performed by the inverse FFT (IFFT).
- There are very fast and efficient implementations of the FFT and IFFT, which is a big reason for the popularity of OFDM.

Fading

- The subcarrier spacing is typically much less than the coherence bandwidth.

Channel is nearly flat for each subcarrier.

Correlator Demodulator Block Diagram (Baseband)

Channel Estimation

- In order to detect the bits, the effects of channel gains must be compensated.
- The subcarrier channel gains are normally estimated using a preamble and pilot tones.
- Simplest approach is to just divide the kth demodulated output by an estimate of

\[ H \left( f_u + \frac{k}{T_u} \right) \]
Preamble Structure

- The Preamble comprises 12 training symbols, 10 short ones and 2 longer ones.

Forward Error-Correction Coding (FEC)

- The information is typically FEC encoded and interleaved prior to modulation.
- The bits carried by faded subcarriers might be detected in error without the FEC.
- With FEC and interleaving, erroneous bits may be correctable, thereby providing frequency diversity.

Modes of DSRC/WAVE

- Modulation and Coding Rate depend on channel quality.

<table>
<thead>
<tr>
<th>Data Rate Mbits/s</th>
<th>Modulation</th>
<th>Coding Rate (R)</th>
<th>Coded Bits per Subcarrier (N_{CBS})</th>
<th>Coded Bits per OFDM Symbol (N_{CBS})</th>
<th>Data Bits per OFDM Symbol (N_{DBPS})</th>
<th>Coded Bits per Subcarrier (N_{CBS})</th>
<th>Coding Rate (R)</th>
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<tbody>
<tr>
<td>3</td>
<td>BPSK</td>
<td>3/2</td>
<td>1</td>
<td>88</td>
<td>24</td>
<td>3</td>
<td>3/2</td>
<td>BPSK</td>
<td>3/2</td>
<td>1</td>
<td>88</td>
</tr>
<tr>
<td>4.5</td>
<td>BPSK</td>
<td>3/4</td>
<td>1</td>
<td>88</td>
<td>36</td>
<td>3</td>
<td>3/4</td>
<td>BPSK</td>
<td>3/4</td>
<td>1</td>
<td>88</td>
</tr>
<tr>
<td>6</td>
<td>QPSK</td>
<td>3/4</td>
<td>2</td>
<td>96</td>
<td>48</td>
<td>3</td>
<td>3/4</td>
<td>BPSK</td>
<td>3/4</td>
<td>2</td>
<td>96</td>
</tr>
<tr>
<td>12</td>
<td>16-QAM</td>
<td>3/4</td>
<td>4</td>
<td>192</td>
<td>96</td>
<td>3</td>
<td>3/4</td>
<td>BPSK</td>
<td>3/4</td>
<td>4</td>
<td>192</td>
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<tr>
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<td>16-QAM</td>
<td>3/4</td>
<td>4</td>
<td>192</td>
<td>144</td>
<td>3</td>
<td>3/4</td>
<td>BPSK</td>
<td>3/4</td>
<td>4</td>
<td>192</td>
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<tr>
<td>24</td>
<td>64-QAM</td>
<td>3/4</td>
<td>6</td>
<td>288</td>
<td>192</td>
<td>3</td>
<td>3/4</td>
<td>BPSK</td>
<td>3/4</td>
<td>6</td>
<td>288</td>
</tr>
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<td>27</td>
<td>64-QAM</td>
<td>3/4</td>
<td>6</td>
<td>288</td>
<td>216</td>
<td>3</td>
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<td>6</td>
<td>288</td>
</tr>
</tbody>
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Summary

- OFDM is a multi-carrier modulation technique.
- Each subcarrier carries BPSK, QPSK or QAM.
- The subcarriers are so close, that each subcarrier sees a flat-faded channel.
- The Guard Interval ensures that successive OFDM symbols do not interfere with each other.
- FEC and interleaving provide frequency diversity.
- OFDM modulation and demodulation is conveniently performed by fast DSP operations (FFT and IFFT).