Coherence Bandwidth

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Correlation in Frequency

- We can view the frequency response of a channel as a Random Process as a function of $f$.
- We can ask, "What is the correlation between responses at different frequencies?"

Review: Correlation Coefficient

- Suppose $X$ and $Y$ are two complex RVs.
- Their correlation coefficient is defined as:
  \[ \rho_{XY} = \frac{E[(X - m_X)(Y - m_Y)^*]}{\sigma_X \sigma_Y} \]
- This is a normalized covariance; it varies between +1 and -1.

Correlation Coefficient for Random Processes

- Now suppose $H(f)$ is a RP wrt $f$.
- That means that for any fixed $f$, $H(f)$ is a RV.
- Consider two frequencies $f_1$ and $f_2$. The correlation coefficient becomes:
  \[ \rho_{H(f_1)H(f_2)} = \frac{E[(H(f_1) - m_{H(f_1)})(H(f_2) - m_{H(f_2)})^*]}{\sigma_{H(f_1)} \sigma_{H(f_2)}} \]

Wide Sense Stationary Uncorrelated Scattering (WSSUS)

- Assumes that path gains at different delays are uncorrelated.
- Assumes correlations between frequency responses depend only on the frequency difference $\Delta f$.
  \[ \rho_{\Delta f} = \frac{E[(H(f) - m_H)(H(f + \Delta f) - m_H)^*]}{\sigma_H^2} \]

Mean is Zero

- Because the phase is uniformly distributed over $[0, 2\pi]$, $m_H = 0$, so the correlation coefficient becomes:
  \[ \rho_{\Delta f} = \frac{E[H(f)H(f + \Delta f)^*]}{E[H(f)^2]} \]
Coherence Bandwidth

- The X% coherence bandwidth is that value of ∆f such that
  \[ \rho_{xy} = \frac{X}{100} \]
- If the 90% coherence bandwidth is 30KHz, then responses for frequencies separated by 30KHz or less will be nearly equal.

Illustration
- Do you think the 90% coherence bandwidth is > or < 100MHz?

Relation to RMS Delay Spread

- The 90% coherence bandwidth is approximately
  \[ B_{c,90} = \frac{1}{50\sigma_r} \]
- The 50% coherence bandwidth is approximately
  \[ B_{c,50} = \frac{1}{5\sigma_r} \]

Need for Equalization

- If a transmitted signal’s bandwidth is greater than the 50% coherence bandwidth, then the channel is frequency selective.
- An equalizer (adaptive tapped delay filter) will be needed in the receiver.
- Flat-fading channels do not require equalization.

Time Dispersion Relationships

- Flat Fading
  - \( B_{c,90} > B_s \)
  - \( \sigma_r < 0.2T_s \)
- Frequency Selective Fading
  - \( B_{c,90} < B_s \)
  - \( \sigma_r > 0.2T_s \)

Summary

- Delay spread and coherence bandwidth are inversely related and quantify the effects of multipath delays.
- They can be used to estimate the maximum data rate that can be supported without the use of an equalizer.
References