Level Crossings and Fade Durations

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Normalized Fading Process
- Begin with the channel fading process, normalized to the local rms signal level

Normalized Threshold Level $\rho$
- Pick a level or threshold $\rho = R / R_{\text{rms}}$, where $R$ is the unnormalized threshold and

$$R_{\text{rms}} = \sqrt{\mathbb{E} \left[ |y(t)|^2 \right]} = \sqrt{\sum_{i=0}^{N} a_i^2}$$

Level Crossing Rate (LCR)
- The LCR at threshold $\rho$ is the expected rate at which the normalized envelope passes the value $\rho$ with a positive slope

Trends
- We expect the highest rate around $\rho \approx 0$ dB, tapering off gently for lower thresholds and abruptly for higher thresholds
- The maximum Doppler frequency just scales the horizontal axis and therefore the rate

LCR For Rayleigh Fading
- For Rayleigh fading and isotropic scattering (Clarke’s Model), the LCR is given by

$$\sqrt{2\pi f_d \rho e^{-\rho^2}}$$

where $f_d$ is the maximum Doppler frequency
LCR For a Ricean Channel

If we assume isotropic scattering plus a non-random component, then the LCR can be approximated as

\[ \sqrt{2\pi(K+1)}f_d \rho e^{-K(K+1)\rho^2} I_0(2\rho\sqrt{K(K+1)}) \]

where \( I_0 \) is the modified Bessel Function of the first kind, zero order:

\[ I_0(x) = \frac{1}{2\pi} \int_0^{\pi} e^{-x \cos \theta} \, d\theta \]

Speed Estimation

The LCR can be used to estimate the speed of a mobile.

Average Fade Duration Impacts Interleaver Depth

The interleaver breaks up the fade so that forward error correction (FEC) codes can correct errors from fading.

De-interleaving

At the receiver, the reverse operation is performed.
Avg. Rayleigh Fade Duration

- For Rayleigh fading and isotropic scattering, the average fade duration below a level $\rho$ is

$$\tau = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_d \rho}$$

Avg. Ricean Fade Duration

- Assuming isotropic scattering with one non-random component,

$$\tau = \frac{1 - Q\left(\sqrt{2K}, \sqrt{2(K+1)\rho^2}\right)}{\sqrt{2\pi}(K+1)f_d \rho e^{-K(K+1)\rho^2/2} \sqrt{K(K+1)}}$$

where $Q(a,b)$ is the Marcum Q function.

Avg. Ricean Fade Duration

- Lines are theoretical results assuming a constant AOA power distribution plus a non-random component.
- Symbols represent simulation results using a multipath fading simulator.

Example

- For a mobile traveling 60 mph, with RF frequency 900MHz, the maximum Doppler frequency is $f_d = 88\text{Hz}$

At the threshold of 0 dB, the average fade rate is 81 fades/s with average duration of 7.8ms

Summary

- Level crossings can be used to estimate mobile speed
- Fade duration must be considered for interleaver design

References