Self-Cancellation of Sample Frequency Offset in OFDM Systems in the Presence of Carrier Frequency Offset

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Abstract - A self-cancellation scheme is proposed to cope with the sample frequency offset (SFO) problem in OFDM systems in the presence of a carrier frequency offset (CFO). Making use of the symmetry between the phase shifts caused by SFO and the subcarrier index, we put the same constellation symbol on symmetrical subcarriers, and combine the pairs at the receiver coherently. In this way, the SFO effects are approximately cancelled with the price of cutting down the bandwidth efficiency by half. However some array gain and diversity gain are obtained from the symmetrical combining. Our scheme can work well together with the phase tracking for residual CFO, so that both SFO and residual CFO can be removed with low complexity. Simulations show that, our scheme effectively removes the effect of SFO; the proposed system outperforms the ideal normal OFDM systems (with no SFO) under the same energy efficiency at high SNR, so the proposed system will also outperform the normal system that using the same overhead for SFO estimation. Finally, a mixed system is proposed to mitigate the drawbacks of our design and the normal OFDM systems for the SFO compensation. Our scheme may be helpful for the synchronization of multiple SFOs in the cooperative transmission.

I. INTRODUCTION

The self-cancellation scheme in this paper is to correct the phase shifts caused by sampling frequency offset (SFO) in the orthogonal frequency-division multiplexing (OFDM) system. The self-cancellation schemes proposed before for the inter-carrier interference (ICI) caused by carrier frequency offset (CFO), IQ imbalance or phase noise [1-3] cannot cancel the effect of SFO. We are the first to apply the self-cancellation method for SFO, which is more difficult to have an accurate estimation than CFO.

OFDM modulation is preferable for high-speed transmission in frequency selective channels. However, because the subcarrier spacing is narrow, an OFDM system is sensitive to the frequency synchronization errors, e.g. CFO and SFO. For packet-based OFDM transmission, CFO cancellation has been well studied in [4-7]. Compared with CFO, SFO is more difficult to estimate accurately. The effect of SFO on the performance of OFDM systems was first addressed by T. Pollet et al [8]. SFO introduces two problems in the frequency domain: inter-channel interference (ICI) and phase rotation of constellations. As mentioned in [8], [11], [13] and [16], the power of the ICI is so small that the ICI are usually taken as additional noise. So the removal of SFO is mainly the correction of phase rotation.

Generally speaking, there are three methods to correct the SFO. The first is to control the sampling frequency of the ADC directly at the receiver [12-14]. However, according to [18], this method is not preferable for low-cost analog front-ends. The second method is interpolation/decimation [9], [11], [15-17]. The SFO is corrected by resampling the base-band signal in the time domain. The problem of this method is that the complexity is too high for high-speed broadband applications. The third method is to rotate the constellations in the frequency domain [12], [13]. The basis for this method is the delay-rotor property [13], which is that the SFO in the time domain causes phase shifts that are linearly proportional to the subcarrier index in the frequency domain. The advantage of this phase rotation method is its low complexity. However, the performance of such method relies on the accuracy of SFO estimation. In previous works, there are three methods for SFO estimation. The first method is cyclic prefix (CP)-based estimation [10]. The performance of this method relies on the length of CP and the delay spread of the multipath channel. The second is the pilot-based method [9], [12] and [17]. The problem with this method is that, because the pilots are just a small portion of the symbol, it always takes several ten’s of OFDM symbols for the tracking loop to converge. The third is the decision-directed (DD) method [14], [16]. The problem of this method is that when SFO is large, the hard decisions are not reliable, so the decisions can be obtained only by decoding and re-constructing the symbol, which requires more memory and complexity. Because no estimation method is perfect, the correction method relying on the estimation will not be perfect. So we consider designing a SFO self-cancellation scheme, so that the effect of SFO can be removed by itself without estimation. Another strong motivation of our work is the synchronization in cooperative transmission (CT). In a CT, multiple SFOs exist at the receiver, so all the methods mentioned above cannot solve the problem. Our SFO self-cancellation scheme is composed of some linear operations, so it will be very helpful for the cancellation of SFOs at the receiver in a CT.

In this paper, we propose a SFO self-cancellation scheme for OFDM systems. In stead of focusing on the linearity between phase shifts caused by SFO and subcarrier index as usual, we make use of the symmetry between them. We put the same constellation on symmetrical subcarriers, and combine each pair coherently at the receiver, so that the phase shifts caused by SFO on symmetrical subcarriers approximately cancel each other. For the phase tracking for residual CFO, pilots are also inserted symmetrically in each OFDM symbol. The drawback of this method is that the bandwidth efficiency is

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cut down by half because of the self-cancellation encoding. However, because of the combination, some diversity is obtained, so the price is decreased. Simulations show that our SFO self-cancellation scheme can work well together with the usual residual CFO correction, and the performance of our system is better than that of the ideal normal OFDM system (with no SFO) under the same energy per bit condition at high SNR. This result tells that, the proposed system will outperform the normal OFDM system in which half of the bandwidth is used as pilots for SFO estimation.

This paper is organized as follows. Section II gives the OFDM signal model. The SFO self-cancellation algorithm is analyzed in Section III. In Section IV, we give simulation results that demonstrate the performance of our algorithm. Section V gives some discussions about our scheme. Finally, conclusions are given in Section VI.

II. OFDM SIGNAL MODEL

A. Basic Model

The width of the inverse fast Fourier transform (IFFT) (or number of subcarriers) is \( N_s \), in which \( N_d \) subcarriers are used for data symbols and \( N_g \) subcarriers are used for pilot symbols. The length of CP is \( N_p \), so the total length of one OFDM symbol is \( N = N_s + N_p \). We use \( f_s \) to denote the sampling frequency, then the sample duration is \( T_s = 1/f_s \). Assuming the constellations of the \( m \)-th OFDM symbol are \( a_{m,k} \) \( (k = -N/2, -N/2+1, \ldots, N/2-1) \) is the subcarrier index), then the transmitted base-band signal is

\[
s(n) = \sum_{m=0}^{M} \sum_{k=-N/2}^{N/2-1} a_{m,k} e^{j2\pi m/n} rect(n-mN_s),
\]

where \( rect(n) = \begin{cases} 1, & 0 \leq n < N_s, \\ 0, & \text{else} \end{cases} \)

Assuming the multipath channel is \( h[n] = \sum_{l=0}^{L_{\text{max}}-1} h_l \delta(n-l) \),

where \( L_{\text{max}} \) is the length of the channel, the received base-band signal of the \( m \)-th OFDM symbol is

\[
r_m[n] = \sum_{l=0}^{L_{\text{max}}-1} h_l s[mN_s + N_p + n-l] + w[n], \quad n = 0, 1, \ldots, N-1,
\]

where \( w[n] \) is the complex Gaussian noise. After FFT, the \( k \)-th subcarrier of the \( m \)-th OFDM symbol is

\[
z_{m,k} = a_{m,k} H_k + w_{m,k},
\]

where \( H_k = \sum_{l=0}^{L_{\text{max}}-1} h_l e^{-j2\pi l/N} \) is the frequency domain response.

B. Introduction of CFO and SFO

We assume \( \Delta f (\text{Hz}) \) is the CFO, \( \epsilon = (T_s' - T_s)/T_s \) is the SFO, and \( T_s' \) is the sample duration of the receiver. According to [9], with the abbreviation \( n' = mN_s + N_p + n \), the received samples can be expressed as

\[
r_m[n] = e^{j2\pi n(1+\epsilon)/N} \sum_{l=0}^{L_{\text{max}}-1} h_l \sum_{k=-N/2}^{N/2-1} a_{m,k} g(n', \epsilon, l) + w[n],
\]

where \( g(n', \epsilon, l) = e^{-j2\pi n'(1+\epsilon)/N} rect(n'(1+\epsilon) - mN_s - l) \).

After FFT, the \( k \)-th subcarrier of the \( m \)-th OFDM symbol is

\[
z_{m,k} = (e^{j2\pi n'/N} e^{j\pi(1+\epsilon)/N} \text{sinc}(\pi/n)) a_{m,k} H_k + \sum_{i \neq k} (e^{j2\pi n'/N} e^{j\pi(1+\epsilon)/N} \text{sinc}(\pi/n)) a_{m,i} H_i + w_{m,k},
\]

with cross-subcarrier and local subcarrier parameters

\[
\phi_{k,i} = (1 + \epsilon)(\Delta f T + i) - k,
\]

\[
\phi_i = \Delta f T + i k,
\]

respectively, where \( T = N T_s' \). In (5), the first term is the expected signal, and the second term is the ICIs from other subcarriers. As mentioned in Section I, after the most part of ICIs is removed by coarse CFO synchronization, the ICIs left can be taken as additional noise \( w_{IC}\). Then the post-FFT signal becomes

\[
z_{m,k} = (e^{j2\pi n'/N} e^{j\pi(1+\epsilon)/N} \text{sinc}(\pi/n)) \text{a}_{m,k} H_k + w_{IC,k} + w_{m,k},
\]

In (7), \( e^{j2\pi n'/N} \) and sinc(\( \pi n/n \)) \( \approx 1 \) are the local phase increment and local amplitude gain, respectively, and they are combined into the estimated channel response \( H_k' = e^{j2\pi n'/N} \text{sinc}(\pi n/n) H_k \).

After channel equalization, (7) becomes

\[
z_{m,k} = e^{-j2\pi n'/N} e^{j\pi(1+\epsilon)/N} a_{m,k} + w_{IC,k} H_k' + w_{m,k}',
\]

where \( w_{IC,k} = w_{IC,k}/H_k' \) and \( w_{m,k}' = w_{m,k}/H_k' \). In (8), only the accumulated phase \( e^{-j2\pi n'/N} \) needs to be corrected.

III. SELF-CANCELLATION OF SAMPLE FREQUENCY OFFSET

A. Idea Demonstration

Our SFO self-cancellation scheme is inspired by the relationship between phase increment and the subcarrier index. Fig. 1 is a simulation result that demonstrates the phase shifts caused by residual CFO and SFO. From the figure, we can find two truths. The first truth is that the phase shifts for the subcarriers in the middle are linearly proportional to the subcarrier index. This is the delay-rotor property mentioned in Section I, and has been explored a lot for estimation and correction of SFO. We also notice that the phase shifts for the edge subcarriers do not obey the linearity. However, for the convenience of design of transmitter and receiver filters, and inter-channel interference suppression, these subcarriers are usually set to be zeros [20]. The other truth is that the phase shifts caused by SFO are symmetrical relative to the constant phase shift caused by residual CFO (dotted horizontal line in Fig. 1), so if we put the same constellation on symmetrical phase shift caused by SFO can approximately cancel each other. This mapping can be called “Symmetric Symbol Repetition (SSR)”, which is different from other self-cancellation techniques, “Adjacent Symbol Repetition (ASR)”, “Symmetric Symbol Repetition (SSR)”, “Adjacent Conjugate Symbol Repetition (ACSR)”, and “Symmetric Conjugate Symbol Repetition (SCSR)”, proposed before in [1-3].
B. Theoretical Analysis

Assuming the same constellation \( a_{m,k} \) is mapped on symmetrical subcarriers \(-k\) and \( k\), the signal (after channel equalization) on the pair can be expressed, according to (8), as

\[
\begin{align*}
    z_{m,k} &= e^{j2\pi(\alpha N_1+m_1)/N} a_{m,k} + w'_{ICL,k} + w'_{m,k}, \\
    z'_{m,k} &= e^{j2\pi(\alpha N_1+m_1)/N} a_{m,k} + w'_{ICL,-k} + w'_{m,-k}.
\end{align*}
\]

Then the sum of \( z_{m,k} \) and \( z'_{m,k} \) is

\[
z_{m,k} = 2\cos(2\pi\alpha G) e^{j2\pi\alpha G T} a_{m,k} + \alpha w'_{ICL,k} + w'_{m,k},
\]

in which \( G = (mN_1 + N_2)/N \). Because \( 2\pi\alpha G \ll 1 \), \( 2\cos(2\pi\alpha G) \approx 2 \). In the same way, because the energy of ICIs is mainly from residual CFO, and the ICIs caused by residual CFO are same for symmetrical subcarriers, the ICIs terms are also combined almost coherently, which means \( \alpha \approx 2 \). So the average SIR does not change after combination. \( w_{m,k} \) and \( w'_{m,k} \) are independent, and the final noise term is

\[
w_{m,k} = w_{m,k} + w'_{m,k} = w_{m,k} |H_k\|^2 + w'_{m,k} |H_k\|^2.
\]

Assuming \( E\{w_{m,k}\} = 0 \), \( E\{|H_k|^2\} = 1 \), \( E\{|w'_{ICL,k}|^2\} = \sigma_{ICL}^2 \), and \( E\{|w'_{m_k}|^2\} = \sigma_\alpha^2 \), under the assumption that \( \sigma_{ICL}^2 << \sigma_\alpha^2 \), the average \( SINR \) before combination (from (7)) and after combination (from (10)) are, respectively,

\[
\begin{align*}
    SINR_{\alpha} &= \frac{1}{\sigma_\alpha^2 + \sigma_{ICL}^2} = \frac{1}{\sigma_\alpha^2}, \\
    SINR_{\alpha} &\approx 4 / (4\sigma_{ICL}^2 + 2\sigma_\alpha^2) = 2 / \sigma_\alpha^2.
\end{align*}
\]

The average \( SINR \) has been improved by 3dB, which is just the array gain from combination. Also, because small values are more likely for \( |H_k| \) than for \( (1/|H_k|^2 + 1/|H_{-k}|^2)^{-1} \), some diversity gain is achieved. Fig. 2 shows that this diversity gain we get is about half of that of 2-branch MRC. In the figure, \( H_1 \), \( H_2 \) and \( H \) are independent Rayleigh fading random variables. Simulations in Section IV will also show the diversity gain.

C. System Structure

Fig. 3 (bottom of this page) gives the structure of the transmitter and receiver with our SFO self-cancellation scheme. At the transmitter, the “Modulation on Half Subcarriers” and “Symmetrical Mapping” blocks compose the Self-Cancellation Encoding module; at the receiver, the “Channel Equalization” and “Symmetrical Combining” blocks compose the Self-Cancellation Decoding module. For the coarse CFO synchronization and channel estimation, repeated short training blocks and repeated long training blocks compose the preamble. To remove the residual CFO, the phase shifts on pilots after the SFO self-cancellation decoding are averaged to get one phase shift, which is multiplied to all the data subcarriers after the self-cancellation decoding.

Fig. 4 shows how to do symmetrical mapping. To do phase tracking for residual CFO correction, pilot symbols are also mapped symmetrically. For the convenience of design of transmit filter and receive filter, twelve subcarriers on the edge are set to be zeros.

![Fig. 4. Symmetrical Mapping.](image)
IV. SIMULATIONS

MATLAB is used for the simulation. Most of the parameters are listed in Table 1.

<table>
<thead>
<tr>
<th>Meaning</th>
<th>Notation</th>
<th>Values in Simulation</th>
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</thead>
<tbody>
<tr>
<td>Total band width</td>
<td>$W$ (Hz)</td>
<td>$8M$</td>
</tr>
<tr>
<td>Number of pilot subcarriers</td>
<td>$N_p$</td>
<td>4</td>
</tr>
<tr>
<td>Number of data subcarriers</td>
<td>$N_d$</td>
<td>48</td>
</tr>
<tr>
<td>Number of zero subcarriers</td>
<td>$N_z$</td>
<td>12</td>
</tr>
<tr>
<td>Total number of subcarriers</td>
<td>$N = N_p + N_d + N_z$</td>
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</tr>
<tr>
<td>Subcarrier Spacing</td>
<td>$d_f$ (Hz)</td>
<td>125k</td>
</tr>
<tr>
<td>Length’ of CP</td>
<td>$L_c$</td>
<td>16</td>
</tr>
<tr>
<td>Length of one complete OFDM symbol</td>
<td>$L = N + L_c$</td>
<td>80</td>
</tr>
<tr>
<td>Number of short training blocks</td>
<td>$N_s$</td>
<td>10</td>
</tr>
<tr>
<td>Length of one short training block</td>
<td>$L_s$</td>
<td>16</td>
</tr>
<tr>
<td>Number of long training blocks</td>
<td>$N_{ls}$</td>
<td>4</td>
</tr>
<tr>
<td>Length of each long training block</td>
<td>$L_l$</td>
<td>80</td>
</tr>
</tbody>
</table>

*Note: the unit of ‘length’ in this table is sample.

½ rate convolutional coding and block interleaving apply to the bits sequence of one packet. The typical urban channel model COST207 [16] is used, and the instantaneous channel gain is normalized to be unity. Each of the short training blocks and each of the long training blocks are defined in the IEEE 802.11a standard [17]. In the simulation, except nine short training blocks and four long training blocks, each packet includes 50 OFDM symbols, so the total length of one packet is 16x10+80x4+80x50 = 4480 samples, which is long enough for most applications. Considering a reasonable value of SFO = 50 ppm [15], the accumulated time offset caused by SFO during the transmission of one packet is 4480x50x10^{-6} = 0.224, which means no sample will be skipped or sampled twice due to SFO. In this case, no Skip/Dup operation is needed.

A. Effectiveness of SFO Self-Cancellation

To show the effectiveness of our SFO self-cancellation, we give the 16QAM constellations in Fig. 5. In the simulation, SNR = 25dB, SFO = 50ppm, and residual CFO exists after the coarse CFO correction. Figures (a) and (c) show the effect of residual CFO and SFO separately. Figure (b) is the ideal case with no SFO and residual CFO is corrected by phase tracking in normal OFDM system. Figure (d) is the result after phase correction (SFO self-cancellation and phase tracking for residual CFO) in our design. From (d) we see that both residual CFO and SFO are removed effectively. In addition, comparing with ideal case of normal OFDM system (b), figure (d) looks even better. This is because we get 3 dB array gain and some diversity gain from the symmetrical combining.

B. Packet Error Rate

To show the performance of the whole system with our SFO self-cancellation scheme, we compare the PER for various simulation configurations in Fig. 6. In the simulations, CFO is 0.3 times the subcarrier spacing, and SFO = 50 ppm. We give two reference systems that do not do SFO self-cancellation: 4QAM, unit transmit power, all pilots in one packet are used for SFO estimation and correction (4QAM-SC-Unit, solid dot-line); 4QAM, unit transmit power with no SFO (4QAM-NSC-Ideal-Unit, dash-line), which is an ideal case. The systems with SFO self-cancellation are: 4QAM, unit transmit power (4QAM-SC-Unit, circle-line); 4QAM, 0.5 transmit power (4QAM-SC-Half, square-line); 16QAM, unit transmit power (16QAM-SC-Unit, triangle-line). Note that 4QAM-SC-Half and 16QAM-SC-Unit have the same energy per bit with 4QAM-NSC-Est-Unit and 4QAM-NSC-Ideal-Unit. From Fig. 6, we can see that, when PER = 0.01: (1) 4QAM-SC-Unit outperforms 4QAM-NSC-Est-Unit by 8 dB, and outperforms 4QAM-NSC-Ideal-Unit by 5.5 dB (3dB array gain and 2.5dB diversity gain); (2) 4QAM-SC-Half outperforms 4QAM-NSC-Est-Unit for all SNRs, and outperforms 4QAM-NSC-Ideal-Unit for high SNR; (3) 16QAM-SC-Unit also outperforms 4QAM-NSC-Est-Unit and 4QAM-NSC-Ideal-Unit.
-Unit at high SNR with the same data rate. Result (1) also tells that, in the normal OFDM system, if half of the bandwidth is used as pilots for SFO estimation (the same bandwidth efficiency with our design), the performance will be much worse than the proposed system (> 5.5dB when PER = 0.01), because the SFO estimation cannot be perfect.

The reason we can get some PER improvement is that we get array gain (if the transmit power is not cut down) and some diversity gain from the self-cancellation decoding. From the figure, we see that F0, F1 and F2 have larger slope than R1 and R2. According to the power delay profile defined in [19], the delay spread \( \sigma_t \approx 50\mu s \), so the 50% coherence bandwidth is \( B_c = 1/(5\sigma_t) \approx 400kHz \), which is about 3.2 times the subcarrier spacing. This means the four subcarriers in the middle are highly correlated, so the two combinations of these two pairs will not bring benefit of diversity, but only array gain if the transmit power is not cut down.

V. DISCUSSION

As mentioned above, our design has two drawbacks. The first one is its low bandwidth efficiency. Although the simulations show that the performance of our design is better than or comparable with the ideal reference system under the same energy per bit at high SNR, we still want to improve the efficiency. The second drawback is that our design has not considered the case when the packet is so long that one sample may be skipped or sampled twice because of the accumulated timing offset. On the other hand, for normal OFDM systems, SFO cannot be estimated accurately during short period of time (this is one of the motivations of our work). So, if we combine our design with the normal OFDM system smartly, the drawbacks of both systems mentioned above can be mitigated.

The mixed system can work in the following way. Because accurate SFO estimation cannot be obtained during the first ten’s of OFDM symbols, these symbols can be transmitted with our SFO self-cancellation scheme. At the receiver, during the decoding of these OFDM symbols, all the phase shifts on pilot subcarriers can be collected together before symmetrical combination for SFO estimation. After the transmission of these ten’s of OFDM symbols, an accurate SFO estimation is already obtained by the receiver, so the system can switch to the normal mode with a SFO tracking loop.

VI. CONCLUSION

A self-cancellation scheme is designed for SFO correction in OFDM systems in the presence of CFO. Making use of the symmetry between phase shifts caused by SFO and subcarrier index, each symmetrical pair of subcarriers carries the same symbol. In the receiver, each pair is combined coherently so that the phase shifts caused by SFO on symmetrical subcarriers approximately cancel each other. In our design, pilots are also inserted symmetrically in each OFDM symbol, so that residual CFO correction by phase tracking can work as usual. Although the bandwidth efficiency of our design is cut down by half because of the self-cancellation encoding, array gain and diversity gain result from the symmetrical combining.

Simulations show that our scheme can work well with normal CFO correction, and the performance of our system is even better than that of ideal normal OFDM system with no SFO under the same energy efficiency at high SNR. Also, our system outperforms the normal OFDM system using the same overhead for SFO estimation. Finally, a mixed system is proposed to compensate the drawbacks of our design and the normal OFDM systems. Because our scheme only involves linear operations, it may be helpful for the correction of multiple SFOs in the cooperative transmission.

REFERENCES