OLA with Transmission Threshold for Strip Networks

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***Abstract*—The opportunistic large array (OLA) with transmission threshold (OLA-T) is a simple form of cooperative transmission that limits node participation in broadcasts. Performance of OLA-T has been studied for disc-shaped networks. This paper analyzes OLA-T for strip-shaped networks. The results also apply to arbitrarily shaped networks that have previously limited node participation to a strip. The analytical results include a condition for sustained propagation, which implies a bound on the transmission threshold. OLA transmission on a strip network with and without a transmission threshold are compared in terms of total energy consumption.**

I. INTRODUCTION

One challenge in wireless ad hoc and sensor networks is energy-efficient routing. By having two or more nodes cooperate to transmit the same packet, cooperative transmission-based strategies offer the spatial diversity benefits of an array transmitter, enabling a dramatic signal-to-noise ratio (SNR) advantage in a multipath fading environment [1], [2]. This advantage can be used to save transmit energy and achieve range-extension [1], [2]. Routing schemes that are based on a form of cooperative transmission (CT) called Opportunistic Large Arrays (OLAs) [3] have been proposed and analyzed for disc-shaped networks [3]–[9]. This paper analyzes the Opportunistic Large Array with Transmission Threshold (OLA-T) for strip-shaped networks. It is shown that as long as the transmission threshold is above a critical value, the packet is delivered to the destination regardless of the distance between the source and the destination.

An OLA is a group of simple, inexpensive relays or forwarding nodes that operate without any mutual coordination, but naturally transmit approximately simultaneously in response to energy received from a single source or another OLA [3]. Each node has just one omnidirectional antenna, however because the nodes are separated in space, the nodes in an OLA collectively provide diversity gain. OLA transmissions avoid individual node addressing, which makes OLA protocols scalable with node density. All the transmissions within an OLA are repeats of the same waveform; therefore the signal received from an OLA has the same model as a multipath channel. Small time offsets (because of different distances and processing times) and small frequency offsets (because each node has a different oscillator frequency) are like excess delays and Doppler shifts, respectively. As long as the receiver, such as a RAKE receiver, can tolerate the effective delay and Doppler spreads of the received signal and extract the diversity, decoding can proceed normally. Alternatively, cooperative diversity can be obtained by combining frequency diversity with power amplifier-friendly modulation schemes such as on-off-shift keying (OOK) and frequency-shift keying (FSK), with a simple energy detector in the receiver. We note that OLA transmission time synchronization with delay spreads less than 300 ns have been demonstrated [5]. Even though many nodes may participate in an OLA transmission, total transmission energy can still be saved because all nodes can reduce their transmit powers dramatically and large fade margins are not needed.

When used for broadcasting in a disc-shaped network, nodes repeat if they haven't repeated the packet before, and the resulting OLAs will "propagate," forming concentric ring shaped OLAs that will eventually include all nodes, under a condition on relay power and receiver sensitivity [3]; we refer to this broadcast scheme as "Basic OLA." OLA with Transmission Threshold (OLA-T) applies an SNR threshold to limit the relaying nodes to those at the edge of the decoding range [4], [7]. Even though these "border nodes" must transmit at a higher power than for Basic OLA to sustain propagation [4], total transmit energy is still saved because nodes

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that are not at the border must waste some of their energy just getting their signals over other nodes that have already received the signal. A CT-based route is a *strip* that is multiple nodes wide. Therefore, a CT-based route can be considered as a "strip network" inside a larger network. A CT-based route can be created by recruiting cooperators near an existing conventional multi-hop route [12], [13]. Another way is by utilizing geographical information (for example, by using global positioning system (GPS)) in [10]. A third way is by using the OLA Concentric Routing Algorithm with Transmission Threshold (OLACRA-T) [14]. Our motivation in the study reported here is to gain an understanding of how OLA-T parameters and the strip width impact route performance. This understanding can help the development of CT-based routing protocols. The analysis of broadcasting using Basic OLA for a strip network was done in [10]. Therefore, this paper represents an extension of [10].

Basic OLA and OLA-T share the important feature that no individual nodes are addressed. Given that the node density is sufficient to sustain OLA transmission, the complexity of these broadcast protocols is absolutely independent of node density, making OLA-based broadcasting very attractive for extremely high density wireless networks. Also, because an OLA-to-OLA "hop" is a transmission between two clusters that each occupy area rather than just one point, OLA-based routes are very robust against mobility [15].

II. System Model

For our analysis, we adopt the notation and assumptions of [4], some of which were used earlier in [10]. Half-duplex nodes are assumed to be distributed randomly and unifromly over a continuous strip with average node density ρ, width W, and length L. Symbolically, let $\mathbb{S} = \{(x, y) : |y| \leq \frac{W}{2},\ 0 \leq x \leq L\}$ denote the network strip. The originating source (assumed to be a point source) and the destination locations are assumed to be at the two ends of the network strip.

We assume a node can *decode and forward* (DF) a packet without error when its received signal-to-noise ratio (SNR) is greater than or equal to a modulation-dependent threshold [4]. Assumption of unit noise variance transforms the SNR threshold to a received power criterion, which is denoted as the decoding or 'lower' threshold, τ_l. We note that the decoding threshold τ_l is not explicitly used in real receiver operations. A real receiver always just tries to decode a packet. If the packet was decoded properly, then it is assumed that the receiver power must have exceeded τ_l. In contrast, the 'transmission' or 'upper' threshold, τ_u is used explicitly in the receiver to compare against the received SNR. This additional criterion for relaying limits the number of nodes in each hop because a node would relay only if it's received SNR is *less* than τ_u. So the thresholds, τ_l and τ_u, define a range of received powers that correspond to the "significant" boundary nodes, which form the OLA. While each boundary node in OLA-T must transmit a somewhat higher power, compared to Basic OLA, there is still an overall transmit energy savings with OLA-T because of the favorable location of the boundary nodes. We define the relative transmission threshold (RTT) as $\mathcal{R} = \frac{\tau_u}{\tau_l}$.

For simplicity, the *deterministic model* [10] is assumed, which means that the power received at a node is the sum of the powers from each of the node transmissions. This implies that signals received from different nodes are orthogonal. The orthogonality can be approximated, for example, with direct sequence spread spectrum (DSSS) modulation, RAKE receivers and by allowing transmitting nodes to delay their transmission by a random number of chips [11].

Following the Continuum Model for a strip network from [10], we assume a non-fading environment and a path-loss exponent of 2. The path-loss function in Cartesian coordinates is given by $l(x, y) = (x^2+y^2)^{-1}$, where (x, y) are the normalized coordinates at the receiver. As in [4], distance d is normalized by a reference distance, d_0. Let the normalized source and relay transmit powers be denoted by P_s and P_r, respectively, and the relay transmit power per unit area be denoted by $\overline{P_r} = \rho P_r$. The normalization is such that P_s and P_r are actually the SNRs at a receiver d_0 away from the transmitter [4]. Since we assume a continuum of nodes in the network, we let the node density ρ become very large ($\rho \to \infty$) while $\overline{P_r}$ is kept fixed. For any finite node density, it can be shown that the node degree as $\mathcal{K} = \pi\overline{P_r}/\tau_l$ [14].

Lastly, following [10], we assume that every node knows whether or not it is a part of the strip, by one of the means mentioned earlier.

III. Cooperative Transmission Protocols for the Strip Network

In this section, broadcasting a source-originated packet over a strip using Basic OLA and OLA-T shall be described.

A. Basic OLA

First, we briefly review a successful strip network broadcast using Basic OLA as preparation for our extension OLA-T. The cooperative transmission protocol is such that the source node, denoted by S in Fig. 1(a), transmits the initial packet. All the nodes in the vicinity of the source node that can decode the packet form the *first* Decoding Level. Next, all the nodes in the *first* Decoding Level transmit the same packet at *approximately* the same time, and constitute the "OLA-1" nodes or the first OLA, denoted by $\mathbb{S}_1$ in Fig. 1(a). Mathematically,

$$\mathbb{S}_1 = \{(x,y) \in \mathbb{S} : P_s \ l(x,y)dxdy \geq \tau_l\}.$$

Next, the set of nodes in the vicinity of OLA 1 that decode the packet but have not previously decoded the same packet, form the *second* Decoding Level. All the nodes in the *second* Decoding Level constitute the second OLA, denoted by $\mathbb{S}_2$ in Fig. 1(a). Mathematically, OLA-2 nodes are given by

$$\begin{aligned}\mathbb{S}_2 =&\{(x,y) \in \mathbb{S} \backslash \bigcup_{i=1}^{1} \mathbb{S}_i : \\ &\overline{P_r} \int\int_{\mathbb{S}_1} l(x-x', y-y')dx'dy' \geq \tau_l\}.\end{aligned}$$

The successive OLAs are defined similarly, and the packet is broadcast over the network. The condition for infinite network broadcast for Basic OLA is given by [10]

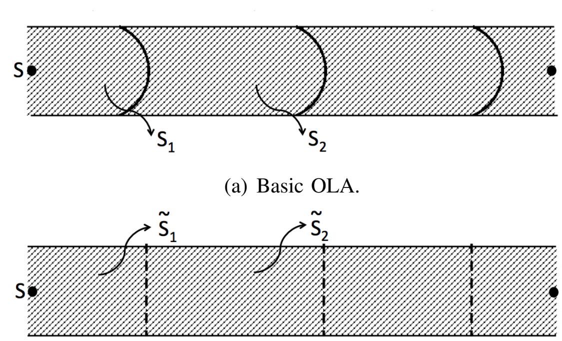
$$2 \geq \exp\left(\frac{1}{\mathcal{K}}\right) \Rightarrow \mathcal{K} \geq \frac{1}{\ln 2}, \tag{1}$$

It is assumed that the nodes do *not* transmit the same packet more than once. The key point is that in Basic OLA, all the nodes in a Decoding Level relay and constitute an OLA.

Fig. 1(a) represents the propagation of a packet along a strip network using Basic OLA, and Fig. 1(b) represents the *straight line* approximation of the curved boundaries. The shaded regions, $\mathbb{S}_1, \mathbb{S}_2, \ldots$ denote the nodes in *each* OLA that were recruited and participated in relaying a packet from the source to the destination. From [10], the OLA-k nodes for Basic OLA are given by

$$\begin{aligned}\mathbb{S}_k =&\{(x,y) \in \mathbb{S} \backslash \bigcup_{i=1}^{k-1} \mathbb{S}_i : \\ &\overline{P_r} \int\int_{\mathbb{S}_{k-1}} l(x-x', y-y')dx'dy' \geq \tau_l\}.\end{aligned} \tag{2}$$

Equation (2) results in curves that are non-linear without closed-form expressions. However, it was shown in [10] that by setting W to be small, the 'curved' boundaries could be approximated by *straight* lines and gave reasonably accurate estimates of network behavior. So the regions $\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3, \ldots$ are approximated by rectangles $\tilde{\mathbb{S}}_1, \tilde{\mathbb{S}}_2, \tilde{\mathbb{S}}_3, \ldots$ as shown in Fig. 1(b).



(a) Basic OLA.

(b) Approximating OLAs with straight lines.

Fig. 1. Propagation along a network strip using Basic OLA with a *straight line* approximation.

B. OLA-T

Next, the OLA-T protocol is described. The source node, S initiates the packet transmission and all the nodes in the vicinity of the source node that can decode the packet form the *first* Decoding Level. However, unlike Basic OLA, *only* the nodes *first* Decoding Level that satisfy the transmission threshold constitute the first OLA, and is denoted by $\mathbb{S}_1$ in Fig. 2. Mathematically,

$$\mathbb{S}_1 = \{(x,y) \in \mathbb{S} : \tau_l \leq P_s \ l(x,y)dxdy \leq \tau_u\}.$$

Next, the set of nodes in the vicinity of OLA 1 that decode the packet, but have not previously decoded the same packet, form the *second* Decoding Level. Again, *only* the nodes in *second* Decoding Level that satisfy the transmission threshold constitute the second OLA, denoted by $\mathbb{S}_2$ in Fig. 2. Mathematically, OLA-2 nodes are given by

$$\begin{aligned}\mathbb{S}_2 =&\{(x,y) \in \mathbb{S} \backslash \mathbb{S}_1 : \\ &\tau_l \leq \overline{P_r} \int\int_{\mathbb{S}_1} l(x-x', y-y')dx'dy' \leq \tau_u\}.\end{aligned}$$

Fig. 2 represents the propagation of a packet along a strip network using OLA with Transmission Threshold (OLA-T). For OLA-T, incorporating the transmission threshold, the OLA-k nodes are given by

$$\begin{aligned}\mathbb{S}_k =&\{(x,y) \in \mathbb{S} \backslash \bigcup_{i=1}^{k-1} \mathbb{S}_i : \\ &\tau_l \leq \overline{P_r} \int\int_{\mathbb{S}_{k-1}} l(x-x', y-y')dx'dy' \leq \tau_u\}.\end{aligned} \tag{3}$$

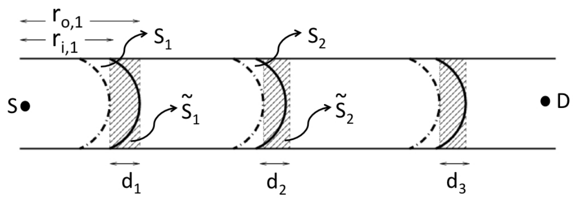


Fig. 2. OLA with Transmission Threshold (OLA-T).

C. Rectangular Approximation

We assume W to be small, so the 'curved' regions (the regions between the solid and dash-dot lines) $\mathbb{S}_i$ are approximated by the 'shaded' rectangles $\tilde{\mathbb{S}}_i$ shown in Fig. 2. With this approximation, we can derive the boundaries for the k-th OLA for the OLA-T protocol. The inner and outer boundaries that define the OLA-1 nodes are $r_{i,1}$ and $r_{o,1}$, respectively (refer to Fig. 2). Using the definition of the path-loss functions defined previously, $r_{i,1} = \sqrt{\frac{P_s}{\tau_u}}$ and $r_{o,1} = \sqrt{\frac{P_s}{\tau_l}}$. $\tilde{\mathbb{S}}_1$ is the first OLA with boundary conditions given by $\sqrt{\frac{P_s}{\tau_u}} \leq x \leq \sqrt{\frac{P_s}{\tau_l}}$ and $|y| \leq \frac{W}{2}$. We denote the length of the first OLA as d_1, given by $d_1 = r_{o,1} - r_{i,1} = \sqrt{\frac{P_s}{\tau_l}} - \sqrt{\frac{P_s}{\tau_u}}$, and height W.

In order to approximate the curved inner and outer boundaries for $\mathbb{S}_2$ by straight lines, $r_{i,2}$ and $r_{o,2}$ are chosen to satisfy

$$\overline{P_r} \int\int_{\tilde{\mathbb{S}}_1} l[x - (r_{i,1} + d_1 + r_{o,2}), y]dxdy = \tau_l, \text{ and} \quad (4)$$

$$\overline{P_r} \int\int_{\tilde{\mathbb{S}}_1} l[x - (r_{i,1} + d_1 + r_{i,2}), y]dxdy = \tau_u, \quad (5)$$

respectively. Applying change of variables to (4), yields

$$\int_{-W/2}^{W/2} \int_{r_{o,2}}^{r_{o,2}+d_1} \frac{\overline{P_r}}{x^2 + y^2} dxdy$$
$$= \int_{r_{o,2}}^{r_{o,2}+d_1} \frac{2\overline{P_r}}{x} \arctan\left(\frac{W}{2x}\right) dx = \tau_l.$$

Similarly,

$$\int_{r_{i,2}}^{r_{i,2}+d_1} \frac{2\overline{P_r}}{x} \arctan\left(\frac{W}{2x}\right) dx = \tau_u.$$

So, $\tilde{\mathbb{S}}_2$ is the second OLA with a length $d_2 = r_{o,2} - r_{i,2}$. In this way, the subsequent OLA lengths $d_3, d_4, \ldots$ can be found iteratively $d_k = r_{o,k} - r_{i,k} = h_o(d_{k-1}) - h_i(d_{k-1})$, where the functions $h_\Omega(d_{k-1})$ for $d_{k-1} > 0$, $\Omega \in \{i, o\}$ are defined as the unique solutions of

$$\int_{h_\Omega(d_{k-1})}^{h_\Omega(d_{k-1})+d_{k-1}} \frac{2\overline{P_r}}{u} \arctan\left(\frac{W}{2u}\right) du = \tau_\Gamma, \quad (6)$$

where $\Gamma = u$ when $\Omega = i$ and $\Gamma = l$ when $\Omega = o$

We denote $h_o(\cdot) - h_i(\cdot) = g(\cdot)$. So, $d_{k+1} = g(d_k)$. In [10], the authors proved that $h(\cdot)$ was monotonically increasing and concave downward, and derived a closed-form expression for $h'(0)$ by knowing the behavior of the integrand of (6), $F(u) = \frac{1}{u}\arctan\left(\frac{1}{2u}\right)$, which was shown to be a decreasing function. Using similar arguments, the following properties for $g(\cdot)$ can be proved semi-analytically.

1) The function g is monotonically increasing.
2) The function g is concave downward.
3) The tangent at zero, $g'(0)$ is given by

$$\begin{aligned} g'(0) &= h_o^{'}(0) - h_i^{'}(0), \\ &= \frac{1}{\exp\left(\frac{1}{\mathcal{K}}\right) - 1} - \frac{1}{\exp\left(\frac{\mathcal{R}}{\mathcal{K}}\right) - 1}. \end{aligned}$$

4) When $g'(0) > 1$, then g has a unique positive fixed point $g(d) = d$. When $g'(0) < 1$, the only fixed point of g is at $d = 0$.

IV. Sufficient and Necessary Conditions for Infinite OLA Propagation

Infinite propagation of the packet is determined by how the sum $\sum_k d_k$ grows with k. When this sum is *infinite*, the OLAs (and hence, the packet) will propagate forever keeping the link between the source and destination intact irrespective of the distance between these points. However, if the sum is *finite*, then the packet does not reach the destination when the source and destination are too far apart. The propagation of the packet along the strip network can be predicted by computing the slope of the *concave* function g at zero, i.e., and this results in two extreme cases:

1) Transmissions die out when $g'(0) < 1$, i.e.,

$$2 < \exp\left(\frac{1}{\mathcal{K}}\right) + \exp\left(\frac{-\mathcal{R}}{\mathcal{K}}\right), \text{ and}$$

2) Transmissions reach a steady state when $g'(0) > 1$, i.e.,

$$2 > \exp\left(\frac{1}{\mathcal{K}}\right) + \exp\left(\frac{-\mathcal{R}}{\mathcal{K}}\right).$$

First, like in [10] we shall prove the condition for the case when the transmissions die out and only a finite portion of the network is reached, i.e., $\lim_{k\to\infty} d_k = 0 \Rightarrow$

$\sum_k d_k < \infty$. Since g is concave downward, the tangent to the curve at $d_k = 0$ stays above, i.e.,

$$g(d_k) \le g'(0)d_k, \ \forall d_k \ge 0.$$

It is desired to upper bound d_{k+1} by $(g'(0))^k d_1$. We establish this using Mathematical Induction. Assume $d_k \le (g'(0))^{k-1} d_1$. So,

$$d_{k+1} = g(d_k) \le g'(0)d_k \le (g'(0))^{k-1} d_1,$$

and hence the upper bound. Consider the sum $\sum_k d_k$.

$$\begin{aligned}\sum_k d_k &\le d_1 \sum_k (g'(0))^k, \\ &= d_1 \frac{1}{1-g'(0)}, \\ &= d_1 \left[\frac{\exp\left(\frac{1+\mathcal{R}}{\mathcal{K}}\right) - \exp\left(\frac{1}{\mathcal{K}}\right) - \exp\left(\frac{\mathcal{R}}{\mathcal{K}}\right) + 1}{\exp\left(\frac{1+\mathcal{R}}{\mathcal{K}}\right) - 2\exp\left(\frac{\mathcal{R}}{\mathcal{K}}\right) + 1}\right], \\ &< \infty.\end{aligned}$$

Since the series is summable, and so $d_k \to 0$ as $k \to \infty$.

Next, we establish the condition for the transmissions reaching a steady value. The convergence of one-dimensional dynamical system can be established by the so-called "staircase diagram" [16] in case there is monotone convergence to a fixed point as shown in Fig. 3. Since g is monotonically increasing and concave downward, when the system starts from an initial condition (d_1 in Fig. 3), which is below the fixed point of g, then d_k increases monotonically towards the attractor or the fixed point. The convergence of the trajectory to a fixed point (defined as the point where the function g and the line $g(d_k) = d_k$ intersect) is determined by the value of the slope, i.e., $|g'(d_k)|$. If $|g'(d_k)| < 1$ at $g(d_k) = d_k$, then the iterate d_k converges to the fixed point. In the example shown in Fig. 3, it takes 5 iterations to reach the fixed point.

Hence, the sufficient and necessary condition for infinite propagation is when

$$2 \ \ge \ \exp\left(\frac{1}{\mathcal{K}}\right) + \exp\left(\frac{-\mathcal{R}}{\mathcal{K}}\right). \tag{7}$$

We observe that this is the same condition as for the infite disc network in [4]. Further, when $\mathcal{R} \to \infty$, OLA-T becomes Basic OLA, and (7) becomes (1), which was derived in [10]. Finally, (7) can be re-written in terms of a lower bound for $\mathcal{R}$ as follows,

$$\mathcal{R}_{\text{lower bound}} = -\mathcal{K} \ln\left[2 - \exp\left(\frac{1}{\mathcal{K}}\right)\right]. \tag{8}$$

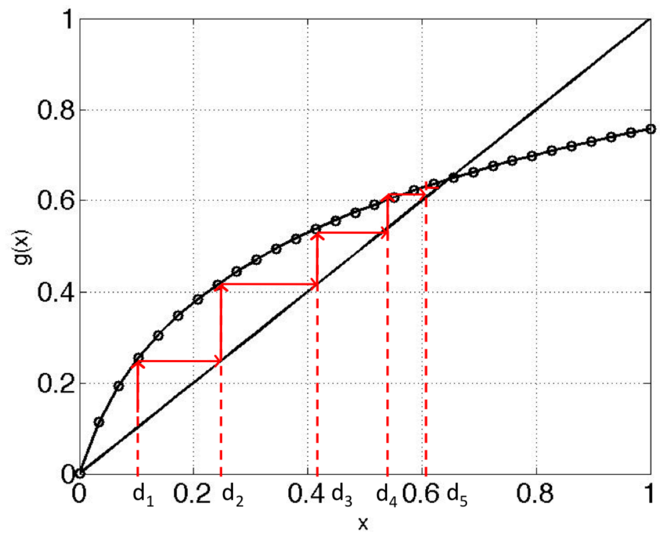


Fig. 3. $g(x)$ versus x for $g'(0) > 1$.

V. Numerical Results and Discussion

Fig. 4 shows the lower bound on RTT, $\mathcal{R}_{\text{lower bound}}$, in dB, versus the node degree, $\mathcal{K}$. It is observed that as $\mathcal{K}$ increases, the 'SNR window' decreases. For example, for $\mathcal{K} = 5$, the minimum transmission threshold is about 1 dB higher than the decoding threshold. It can also be inferred that theoretically, it is possible for OLA-T to achieve infinite network broadcast with an infinitesimally small $\mathcal{R}_{\text{lower bound}}$ and very high $\mathcal{K}$.

Fig. 5 shows the different broadcast scenarios depending on the value of the slope at $x = 0$. To generate these results, a node degree, $\mathcal{K} = \pi$ was assumed, which resulted in $\mathcal{R}_{\text{lower bound}} = 1.476$ or 1.68 dB. $\mathcal{R}$ was chosen to be 1.3 (1.13 dB) and 3 (4.77 dB) for the cases, $g'(0) < 1$ and $g'(0) > 1$, respectively. So, $g'(0) < 1 \Rightarrow \mathcal{R} < \mathcal{R}_{\text{lower bound}}$, which results in very thin OLAs (fewer nodes) that are too weak to sustain infinite propagation and eventually die out. This is denoted by the dotted line in Fig. 5. The other extreme is when $g'(0) < 1 \Rightarrow \mathcal{R} > \mathcal{R}_{\text{lower bound}}$, and this is represented by the dashed lines in Fig. 5. It can be seen that the transmissions reach a steady state with the limiting OLA length, $d_\infty > 0$, i.e., a fixed-point attractor away from zero, ensuring that the transmissions don't die out.

Like in [4], we use the fraction of transmission energy saved (FES) as the mertic for comparing the energy-efficiency of OLA-T relative to Basic OLA. However, the FES for the strip network is computed as follows. We use the Fraction of *Transmission* Energy Saved (FES) as the mertic for comparing the energy-efficiency of OLA-T relative to Basic OLA. The FES for the strip network

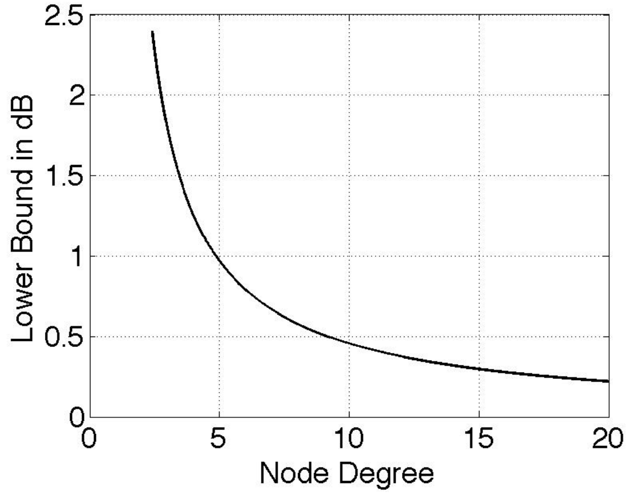


Fig. 4. Lower bound on RTT, $\mathcal{R}_{\text{lower bound}}$, in dB versus node degree, $\mathcal{K}$.

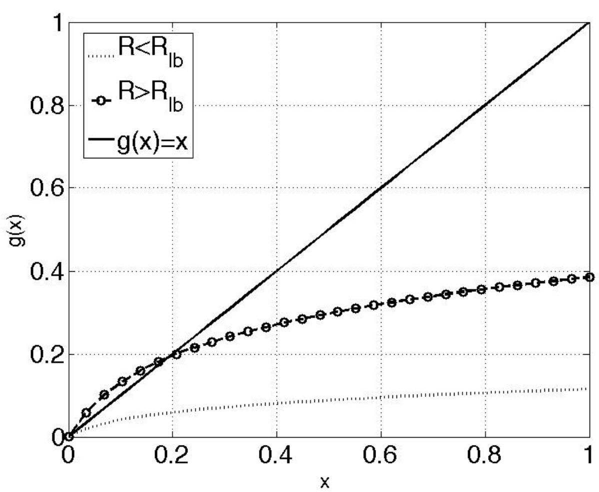


Fig. 5. $g(x)$ versus x for the three cases; $g'(0) < 1$ and $g'(0) > 1$.

is computed as follows. The energy consumed by OLA-T in the first N levels is mathematically expressed as $\overline{P_r}_{(\text{OT})} T_s W \sum_{k=1}^{N} d_k$, where $\overline{P_r}_{(\text{OT})}$ is the lowest value of $\overline{P_r}$ that would guarantee successful broadcast using OLA-T and T_s is the length of the packet in time units. The energy consumed by Basic OLA will be $\overline{P_r}_{(\text{O})} T_s W \sum_{k=1}^{N} r_{o,k}$, where $\overline{P_r}_{(\text{O})}$ is the lowest value of $\overline{P_r}$ that would guarantee successful broadcast using Basic OLA. So, FES can be expressed as

$$\text{FES} = 1 - \frac{\overline{P_r}_{(\text{OT})} T_s W \sum_{k=1}^{N} d_k}{\overline{P_r}_{(\text{O})} T_s W \sum_{k=1}^{N} r_{o,k}} = 1 - \frac{\mathcal{K}_{(\text{OT})} \ln 2 \sum_{k=1}^{N} d_k}{\sum_{k=1}^{N} r_{o,k}}, \tag{9}$$

where $\mathcal{K}_{(\text{OT})}$ is the minimum node degree for OLA-T to guarantee successful broadcast when operating in it's minimum power configuration.

FES defined in (9) is in terms of only the transmit energy, and can be rewritten as $\text{FES} = 1 - \frac{\text{TT}}{\text{TB}}$, where TT is the total transmit energy of our broadcasting algorithm (e.g. OLA-T) and TB is the transmit energy of the Basic OLA. Let the total receive energy (RE) consumed by the network be proportional to TB: $\text{RE} = \alpha\text{TB}$. For example, if the receive energy is the same as transmit energy, then $\alpha = 1$. Then, we can define the whole energy fraction of energy saved (WFES) as follows:

$$\begin{aligned} \text{WFES} &= 1 - \left(\frac{\text{TT} + \text{RE}}{\text{TB} + \text{RE}}\right) \\ &= \left(\frac{1}{1+\alpha}\right)\left(1 - \frac{\text{TT}}{\text{TB}}\right) = \frac{\text{FES}}{1+\alpha}. \end{aligned} \tag{10}$$

Fig. 6 shows FES versus minimum node degree, $\mathcal{K}_{(\text{OT})}$ for a strip network for different values of α. We note that when $\alpha = 0$, we only consider the transmit energy, and when $\alpha \neq 0$, the receive energy is some fraction of the transmit energy. For example, for $\alpha = 0$, at $\mathcal{K}_{(\text{OT})} = 20$, FES is about 0.58. This means that at their respective lowest energy OLAs (OLA-T at $\overline{P_r}_{(\text{OT})}$, and Basic OLA at $\overline{P_r}_{(\text{O})}$), OLA-T saves about 58% of the transmit energy used by Basic OLA at this $\mathcal{K}_{(\text{OT})}$. On the other hand, when both the receive and transmit energies are considered, for for $\alpha = 1$ and $\mathcal{K}_{(\text{OT})} = 20$, the WFES is about 0.29. This means that at their respective lowest energy levels (OLA-T at $\overline{P_r}_{(\text{OT})}$, and Basic OLA at $\overline{P_r}_{(\text{O})}$), OLA-T saves about 29% of the total energy consumed during broadcast by Basic OLA at this $\mathcal{K}_{(\text{OT})}$. We remark that the minimum node degree required for successful broadcast using Basic OLA is 1.44, which is also the lowest possible node degree for OLA-T.

VI. Conclusions

In this paper, we analyzed a simple form of cooperative transmission protocol, Opportunistic Large Array with Transmission Threshold (OLA-T), for broadcasting along a strip network. The wireless network was approximated as a continuum of nodes for the purpose of analysis. It was shown that as long as the transmission threshold is above a critical value, the packet is delivered

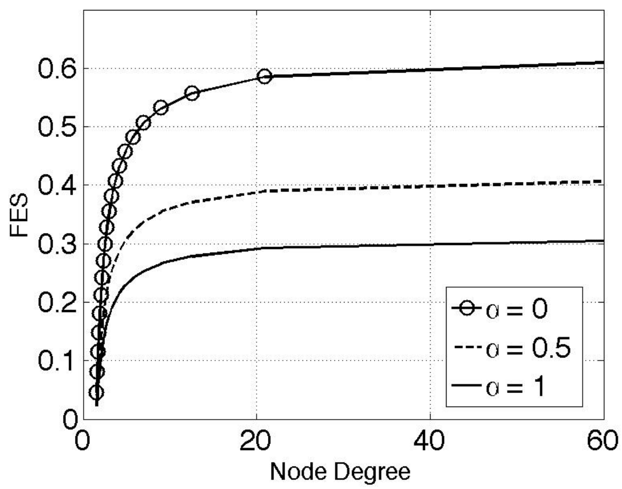


Fig. 6. Variation of FES with the minimum node degree, $\mathcal{K}_{(\mathrm{OT})}$.

to the destination regardless of the distance between the source and the destination. OLA-T protocol limits the node participation in a strip and hence, saves transmission energy. For an example, OLA-T was found to save as much as 61% of the transmitted energy relative to Basic OLA, when both protocols operated in their lowest power configurations. In this paper, however, only a single flow was considered, i.e., a single source-initiated transmission. Analysis of multiple source transmissions will be more complicated and involves considering issues such as collisions and medium access. Future work includes a consideration of the random network and for different path-loss exponents.

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